

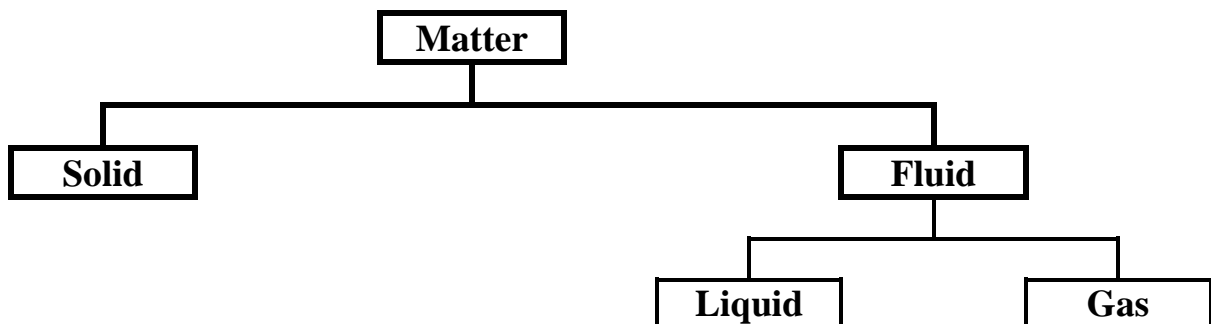
Chapter 1

INTRODUCTION

1-1 The Fluid. 1-2 Dimensions. 1-3 Units. 1-4 Fluid Properties.

1-1 The Fluid:

It is the substance that deforms continuously when subjected to a shear stress.



1-2 Dimensions:

Term	Symbol	Dimensions	
		M-L-T	F-L-T
Length	L	L	L
Area	A	L ²	L ²
Volume	V	L ³	L ³
Time	t	T	T
Velocity	v	L/T	L/T
Angular Velocity	ω	T ⁻¹	T ⁻¹
Acceleration	a	L/T ²	L/T ²
Rate of Discharge	Q	L ³ /T	L ³ /T
Mass	M	M	FT ² /L
Force	F	ML/T ²	F

Term	Symbol	Dimensions	
		M-L-T	F-L-T
Density	ρ	M/L ³	FT ² /L ⁴
Specific Weight	γ	M/L ² T ²	F/L ³
Dynamic Viscosity	μ	M/LT	FT/L ²
Kinematic Viscosity	ν	L ² /T	L ² /T
Pressure	P	M/LT ²	F/L ²
Momentum (or Impulse)	M (or I)	ML/T	FT
Energy (or Work)	E (or W)	ML ² /T ²	FL
Power	p	ML ² /T ³	FL/T
Surface Tension	σ	M/T ²	F/L

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1-3 Units:

Unit System	L	T	M	F
Metric System	cm	sec	gm	Dyne
English System	ft	sec	lb	pdl.
American System	ft	sec	slug	lb.force
S. I. System	m	sec	kg	Newton

Length: 1 ft = 30.48 cm
 1 cm = 0.0328 ft

1 mile = 1609 m

1 mile = 1760 yard

1 yard = 3 ft

1 ft = 12 inch

1 inch = 2.54 cm

Mass: 1 lb = 453.6 gm
 1 slug = 32.2 lb

Force: 1 lb.force = 32.2 pdl
 1 lb.force = 4.445 N

$$1 \text{ Newton} = 10^5 \text{ Dyne} = 10^5 \text{ gm.cm/sec}^2 = 1 \text{ kg.m/sec}^2$$

Gravitational or engineering units of force are gm.force and kg.force.

Absolute or scientific (SI) units are dyne and Newton.

Gravitational units of force = "g" times SI units.

$$g = 981 \text{ cm/sec}^2 = 9.81 \text{ m/sec}^2 = 32.2 \text{ ft/sec}^2$$

$$1 \text{ gm.force} = 981 \text{ Dyne} \quad \& \quad 1 \text{ kg.force} = 9.81 \text{ Newton}$$

$$1 \text{ Newton} = 10^5 \text{ dyne} \quad \& \quad 1 \text{ dyne} = 1 \text{ gm.cm/sec}^2$$

Pressure: 1 bar = 10^5 Pa = 10^5 N/m²
 1 bar = 14.7 psi (lb/in²)
 1 bar = 1.02 kg/cm²

Power: 1 hp = 745.7 W

1-4 Fluid Properties:

Density (ρ):

It is the mass per unit volume.

$$\rho = \frac{M}{V}$$

$$\rho \text{ for water} = 1 \text{ gm/cm}^3 = 1000 \text{ kg/m}^3 \quad \& \quad \rho \text{ for air} = 1.205 \text{ kg/m}^3$$

Specific Weight (γ):

It is the weight per unit volume.

$$\gamma = \frac{W}{V} = \frac{Mg}{V} = \rho g$$

$$\gamma \text{ for water} = 1 \text{ gm/cm}^3 = 1 \text{ t/m}^3 = 1000 \text{ kg/m}^3$$

$$\begin{aligned}\gamma \text{ for water} &= 62.4 \text{ lb/ft}^3 \\ &= 981 \text{ dyne/cm}^3 = 9810 \text{ N/m}^3\end{aligned}$$

$$\gamma \text{ for mercury} = 13546 \text{ kg/m}^3$$

$$\gamma \text{ for air} = 1.23 \text{ kg/m}^3$$

Specific Volume (S.V.):

It is the volume per unit mass.

$$\text{S.V.} = \frac{V}{M} = \frac{1}{\rho}$$

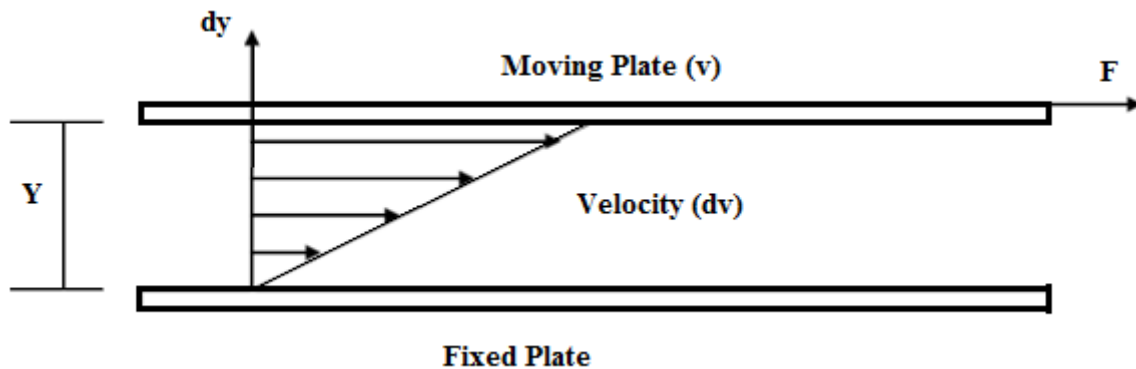
Specific Gravity (S.G.):

It is the ratio of the density of a liquid (or a gas) to the density of the water (or the air).

$$\text{S.G.} = \frac{\rho \text{ fluid}}{\rho \text{ water}} = (\text{mathematically}) \frac{\gamma \text{ fluid}}{\gamma \text{ water}}$$

Viscosity:

It is the resistance of motion or translation of one layer of the liquid relative to other. It is due to cohesion between the particles of a moving fluid.



Newton's law of viscosity: "The shear stress on a layer of a fluid is directly proportional to the rate of shear strain".

$$\tau \propto \frac{dv}{dy} = \mu \frac{dv}{dy}$$

$$\mu = (F/A) / (v/L) = (F/A) / (1/T) = (F \times T) / A = (MLT^{-2} \times T) / L^2 = M / L T$$

τ : Shear stress.

μ : Viscosity or dynamic viscosity.

dv/dy : Shear strain = rate of angular deformation & Unit: rad/sec

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$$\text{Poise} = \text{Dyne} \cdot \text{sec} / \text{cm}^2 = 0.1 \text{ kg} / \text{m} \cdot \text{sec}$$

$$1 \text{ Poise} = 10^{-5} \text{ N} \cdot \text{sec} / \text{cm}^2 = 0.1 \text{ N} \cdot \text{sec} / \text{m}^2 = 0.1 \text{ Pa} \cdot \text{sec}$$

$$\mu \text{ for water} = 1.14 \times 10^{-3} \text{ kg} / \text{m} \cdot \text{sec}$$

$$\mu \text{ for mercury} = 1.552 \text{ kg} / \text{m} \cdot \text{sec}$$

$$\mu \text{ for air} = 1.78 \times 10^{-5} \text{ kg} / \text{m} \cdot \text{sec}$$

Kinematic Viscosity (ν):

$$\nu = \mu / \rho$$

$$\text{Stoke} = \text{cm}^2 / \text{sec} = 10^{-4} \text{ m}^2 / \text{s}$$

$$\nu \text{ for water} = 1.14 \times 10^{-6} \text{ m}^2 / \text{sec}$$

$$\nu \text{ for mercury} = 1.145 \times 10^{-4} \text{ m}^2 / \text{sec}$$

$$\nu \text{ for air} = 1.46 \times 10^{-5} \text{ m}^2 / \text{sec}$$

Example 1:

When 5.6 m^3 of oil weighs 46800 N, find its density and its specific gravity?

Solution

$$W = M \times g$$

$$M = 46800 / 9.81 = 4770.6 \text{ kg}$$

$$\therefore \rho = M / V = 4770.6 / 5.6 = 852 \text{ kg} / \text{m}^3$$

$$\text{S.G.} = \rho (\text{oil}) / \rho (\text{water})$$

$$\rho (\text{water}) = 1000 \text{ kg} / \text{m}^3$$

$$\therefore \text{S.G.} = 852 / 1000 = 0.852$$

Example 2:

When the viscosity of water is 0.00995 Poise, find its value in Pa.sec?

Determine the kinematic viscosity in Stoke?

Solution

$$\mu = 0.00995 \times 0.1 = 9.95 \times 10^{-4} \text{ N.sec/m}^2$$

$$\therefore \mu = 9.95 \times 10^{-4} \text{ Pa.sec}$$

$$v = \mu / \rho$$

$$\mu = 9.95 \times 10^{-4} \text{ kg/m.sec} \quad \& \quad \rho = 1000 \text{ kg/m}^3$$

$$v = (9.95 \times 10^{-4}) / 1000 = 9.95 \times 10^{-7} \text{ m}^2/\text{sec}$$

$$\therefore v = (9.95 \times 10^{-7}) \times 10^4 = 9.95 \times 10^{-3} \text{ Stoke}$$

Example 3:

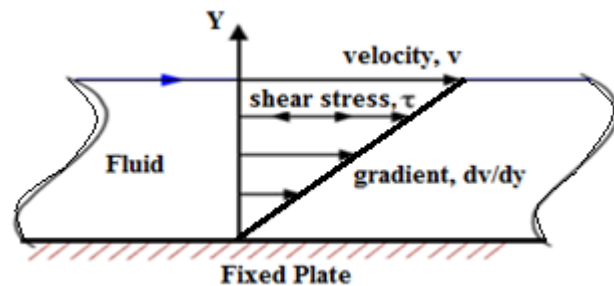
The velocity is 1.125 m/s at a distance of 75 mm from the boundary. The fluid has viscosity 0.048 Pa.s and a relative density 0.913.

Assuming a linear velocity distribution, what are the velocity gradient and the shear stress?

Determine the kinematic viscosity?

Solution

$$\frac{dv}{dy} = \frac{1.125}{0.075} = 15 \text{ sec}^{-1}$$



$$\tau = \mu (dv/dy) = 0.048 \times 15 = 0.72 \text{ Pa}$$

$$v = \mu / \rho$$

$$\rho = 0.913 \times 1000 = 913 \text{ kg/m}^3 \quad (\text{SI system})$$

$$\therefore v = 0.048 / 913 = 5.257 \times 10^{-5} \text{ m}^2/\text{sec}$$

Chapter 2**STATICS OF FLUIDS**

2-1 Fluid Pressure

2-3 Total Pressure on Inclined Submerged Surfaces

2-5 Total Pressure on Curved Surfaces

2-2 Measuring Fluid Pressure

2-4 Centre of Pressure

2-6 Buoyancy

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2-1 Fluid Pressure:

Pressure is the force per unit perpendicular area.

$$P = \frac{F}{A}$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 10^5 \text{ N/m}^2$$

$$1 \text{ bar} = 14.7 \text{ psi (lb/in}^2\text{)}$$

$$1 \text{ bar} = 1.02 \text{ kg/cm}^2$$

Pressure can also be expressed as a height of equivalent liquid column.

$$P = \gamma h$$

If $P = 2 \text{ kg/cm}^2$, then $P = 2000 \text{ gm/cm}^2 = \gamma h = 1 \times h$

$\therefore h = 2000 \text{ cm} = 20 \text{ m of water column.}$

If $P = 20 \text{ kN/m}^2$, then $P = 20000 \text{ N/m}^2 = \gamma h = 9810 \times h$

$\therefore h = 20000 / 9810 = 2.04 \text{ m of water column.}$

Pascal's Law: “The intensity of pressure is the same in all directions at any point in a fluid at rest”.

“The pressure is the same at any two points in the same elevation in a continuous mass of a fluid at rest”.

Atmospheric Pressure: It is the pressure exerted by a column of air of 1 cm² cross sectional area, and a height equal to that of the atmosphere at the sea level.

$$\begin{aligned} \text{At the sea level, } P_{\text{atm}} &= 1.03 \text{ kg/cm}^2 \\ &= 10.3 \text{ m of water} \\ &= 76 \text{ cm of mercury.} \end{aligned}$$

Gauge Pressure: It is the measured pressure, taking P_{atm} as datum.

Absolute Pressure: It is the algebraic sum of the atmospheric pressure and the gauge pressure.

$$P_{\text{abs}} = P_{\text{atm}} + P_g$$

2-2 Measuring Fluid Pressure:

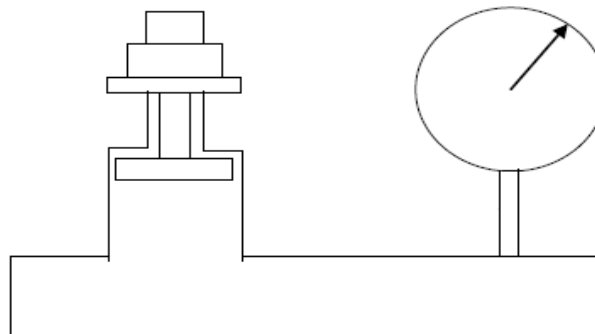
A) Mechanical Gauges:

Dead Weight Pressure Gauges:

It consists of a piston on a cylinder with known area, and it is connected to the gauge point (in a pipe containing a pressed fluid for example) by a tube.

The pressure P is: $P = \text{Weight} / \text{Area of Piston}$

This gauge is suitable for measuring very high pressures.



Example 1:

A dead weight pressure gauge is used to measure the pressure of a liquid in a pipe at the same level. The value of the weight is 8500 N.

1. If the area of the piston is 100 cm², determine the pressure in the pipe?
2. If the piston is 0.5 m below the pipe, determine the pressure in the pipe?

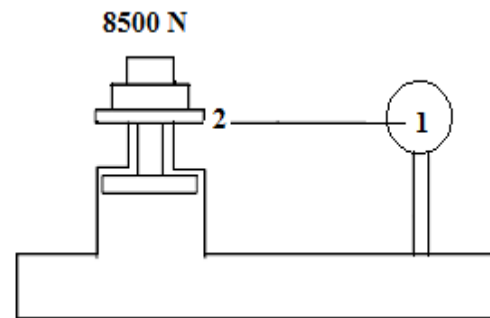
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Solution

1. At the same level:

$$P_1 = P_2$$

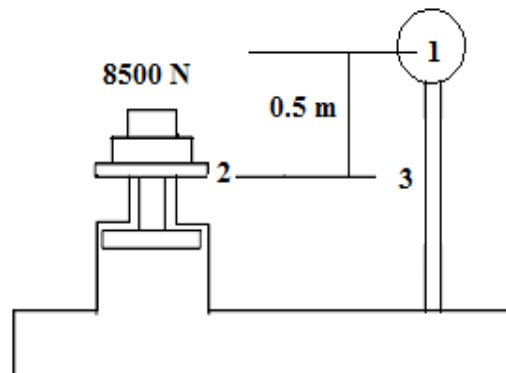
$$\therefore P_1 = W/A = 8500/100 = 85 \text{ N/cm}^2$$



2. At different levels:

$$P_3 = P_1 + \gamma h = P_2$$

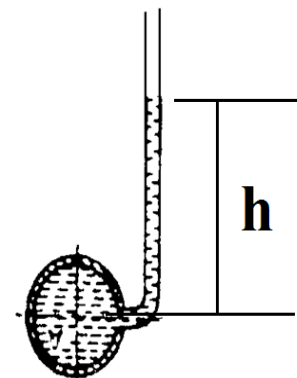
$$\begin{aligned} \therefore P_1 &= P_2 - \gamma h \\ &= 85 - (981 \times 10^{-5} \times 50) \\ &= 84.51 \text{ N/cm}^2 \end{aligned}$$

**B) Tube Gauges:****I- Piezometer Tube:**

It is an open tube, which one end is open to the atmosphere, and the other end is attached to the gauge point (in a pipe containing a pressed liquid for example). The height of the liquid in the tube gives the head pressure.

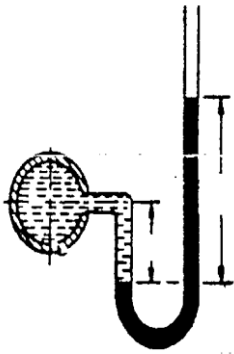
It is used for measuring moderate pressures.

The piezometer tube is not suitable for measuring negative pressure; otherwise the air will enter in the pipe through the tube.

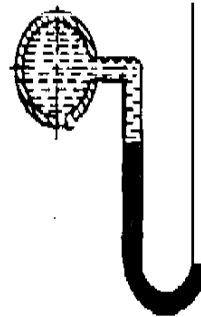


II- Manometer:

It is a U shape tube with a liquid (mercury in general). One end of the tube is open to the atmosphere, and the other end is attached to the gauge point. It is suitable for measuring both high and negative pressures.

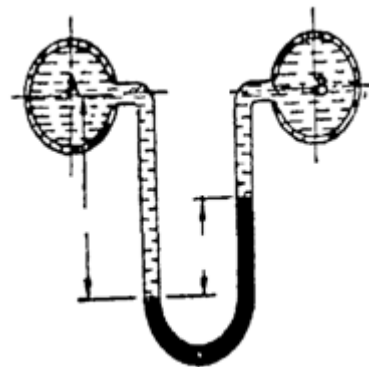
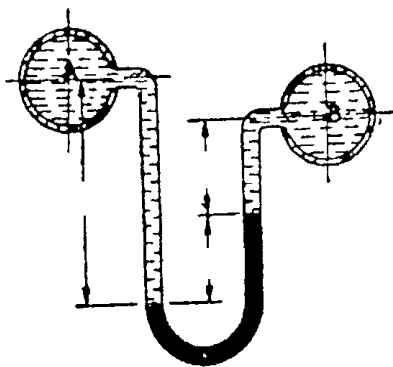


Positive Pressure

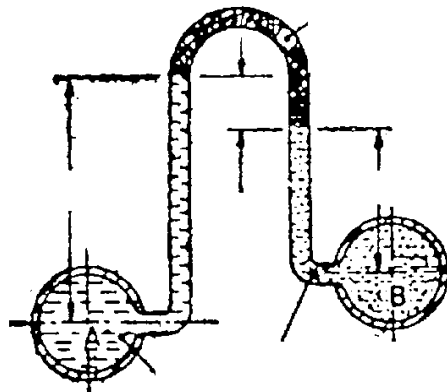


Negative Pressure

Simple Manometer



Differential Manometer



Inverted Differential Manometer

Example 2:

A simple manometer with mercury is used for measuring the pressure of oil in a pipe line, as shown in figure. The specific gravity is 0.8 for oil, and is 13.6 for mercury.

Determine the absolute pressure of oil in the pipe in kg/cm²?

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Solution

At the datum, $P_L = P_R$

$$P + (\gamma_o \times 6) = (\gamma_m \times 15)$$

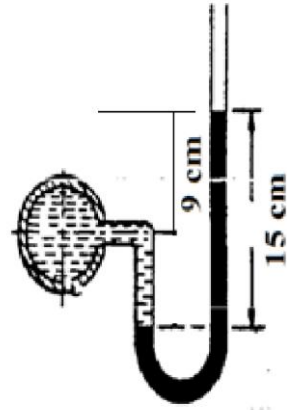
$$\gamma_o = SG_o \times \gamma_w = 0.8 \times 0.001 = 0.0008 \text{ kg/cm}^3$$

$$\gamma_m = SG_m \times \gamma_w = 13.6 \times 0.001 = 0.0136 \text{ kg/cm}^3$$

$$P = (\gamma_m \times 15) - (\gamma_o \times 6) = 0.2 \text{ kg/cm}^2$$

$$P_{abs} = P_{atm} + P_g$$

$$\therefore P_{abs} = 1.03 + 0.2 = 1.23 \text{ kg/cm}^2$$

**Example 3:**

A simple manometer with mercury is used for measuring the pressure of water in a pipe line. Mercury is 13.6 specific gravity.

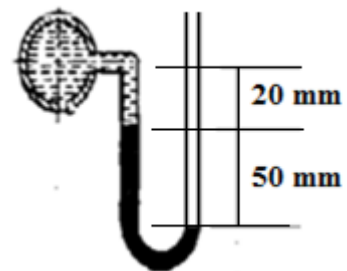
Determine the pressure in the pipe?

Solution

At the datum, $P_L = P_R$

$$P + (\gamma_w \times 20) + (\gamma_m \times 50) = 0$$

$$\therefore h = -20 - (13.6 \times 50) = -700 \text{ mm of water}$$

**Example 4:**

A differential manometer is connected between two points A and B in a pipe of oil (specific gravity = 0.8). The reading (difference in mercury levels) is 100 mm.

Determine the difference in pressures between A and B in terms of head of water, and gm/cm²?

Solution

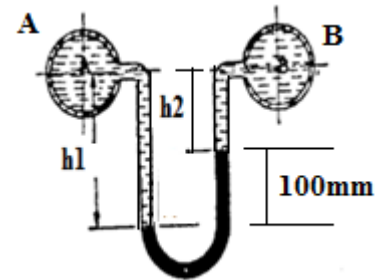
At the datum, $P_L = P_R$

$$P_A + (\gamma_o \times h_1) = P_B + (\gamma_o \times h_2) + (\gamma_m \times 100)$$

$$\begin{aligned} P_A - P_B &= \gamma_o \times (h_2 - h_1) + \gamma_m \times (h_1 - h_2) \\ &= (h_1 - h_2) \times (\gamma_m - \gamma_o) \end{aligned}$$

$$\therefore h_A - h_B = h (S.G_m - S.G_o) = 10 (13.6 - 0.8) = 128 \text{ cm of water}$$

$$\& P_A - P_B = \gamma_w \times h = 1 \times 128 = 128 \text{ gm/cm}^2$$



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Example 5:

An inverted differential manometer with oil of specific gravity of 0.75 is connected to two pipes of water A&B. The pressure at A is 1.5 m of water.

Determine the pressure in the pipe B?

Solution

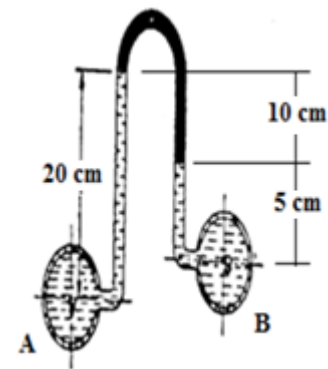
At the datum, $P_L = P_R$

$$P_A - (\gamma_w \times 20) = P_B - (\gamma_w \times 5) - (\gamma_o \times 10)$$

$$(P_A / \gamma_w - P_B / \gamma_w) = (1 \times 20) - (1 \times 5) - (0.75 \times 10)$$

$$P_B / \gamma_w = 150 - 20 + 5 + 7.5 = 142.5 \text{ cm of water}$$

$$\therefore P_B = \gamma_w \times h = 1 \times 142.5 = 142.5 \text{ gm/cm}^2$$

**2-3 Total Pressure on Inclined Submerged Surfaces:**

For a fluid at rest, there are no shear stresses. The pressure forces from the fluid are normal to the exposed surfaces.

The pressure force is equal to the multiplication of the pressure by the area of the surface.

For a submerged plane inclined surface in a liquid, as shown in the figure.

θ : Angle between inclined surface and liquid top surface.

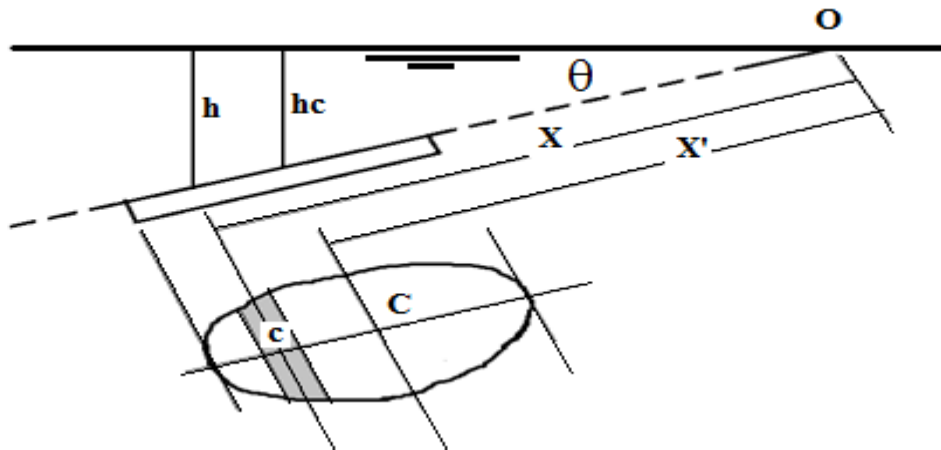
h : Depth of centre of gravity (C) of element below liquid top surface.

h_c : Depth of centre of gravity (C) of surface below liquid top surface.

X : Distance from O to C of the element.

X' : Distance from O to C of the surface.

γ : Specific weight of the liquid.



Pressure on element $P = \gamma h = \gamma (x \sin \theta)$

Area of the element = dA

Pressure force on the element $dF = P dA = \gamma x \sin \theta dA$

Total pressure force on the surface $F = \int dF$

$$F = \int \gamma x \sin \theta dA = \gamma \sin \theta \int x dA$$

$$\int x dA = x' A = \text{First moment of area}$$

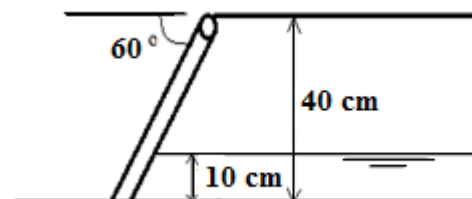
$$F = \gamma x' \sin \theta A = \gamma h_c A = P' A$$

P' : Pressure at C of the surface.

Example 6:

A passage 40 cm x 40 cm is covered at its outlet by a gate that is inclined at 60° with horizontal, and is hinged at the upper edge. The depth of water in the passage is 10 cm.

Determine the pressure force on the gate?



Solution

$$F = \gamma h_c A \quad \gamma = 1 \text{ gm/cm}^3$$

$$h_c = 5 \text{ cm}$$

$$A = (10 / \sin 60) \times 40 = 461.88 \text{ cm}^2$$

$$\therefore F = 1 \times 5 \times 461.88 = 2309 \text{ gm}$$

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Example 7:

A rectangular tank 5m long and 2 m wide contains water 2.5 m deep.
Determine the pressure force on the base of the tank?

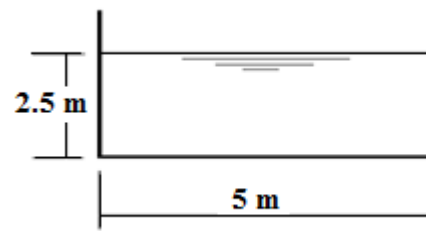
Solution *Total Pressure on Horizontal Submerged Surfaces*

$$F = \gamma h_c A \quad \gamma = 1 \text{ t/m}^3$$

$$h_c = 2.5 \text{ m}$$

$$A = 5 \times 2 = 10 \text{ m}^2$$

$$\therefore F = 1 \times 2.5 \times 10 = 25 \text{ ton}$$

**Example 8:**

A circular gate 1.0 m diameter closes a hole in a vertical retaining wall against sea water that has a specific gravity of 1.03. The centre of the hole is 2.0 m below the water top surface.
Determine the pressure force on the gate?

Solution *Total Pressure on Vertical Submerged Surfaces*

$$F = \gamma h_c A$$

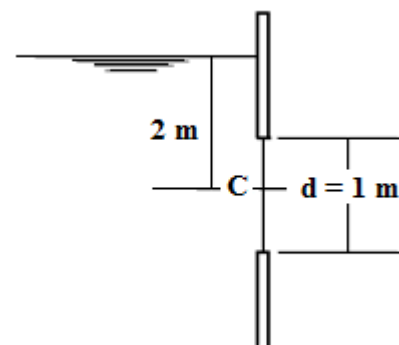
$$\text{S.G} = 1.03 = \gamma / \gamma_w$$

$$\gamma = 1 \times 1.03 = 1.03 \text{ t/m}^3$$

$$h_c = 2 \text{ m}$$

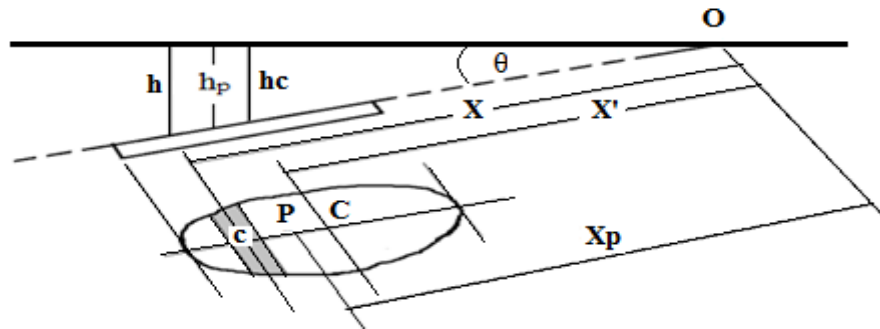
$$A = \pi d^2 / 4 = (3.14 \times 1^2) / 4 = 0.785 \text{ m}^2$$

$$\therefore F = 1.03 \times 2 \times 0.785 = 1.62 \text{ t}$$



2-4 Centre of Pressure:

For horizontal submerged surface, the pressure is constant over the surface. Then, the pressure force acts at centre of gravity (C) of the surface, and its line of action is normal to the area of the surface.



For not horizontal submerged surface, the pressure is linearly distributed over the surface. Then, the centre of pressure P is below the centre of gravity (C) of the surface, due to the linear variation of pressure with the depth.

Taking moments of the pressure forces of element about O:

$$M = \int dF y = \int (\gamma h dA) y = \int (\gamma y \sin \theta dA) y = \int \gamma y^2 \sin \theta dA = \gamma \sin \theta \int y^2 dA$$

$$\int y^2 dA = 2^{nd} \text{ moment of area} = I_o = \text{Moment of inertia of the surface about O}$$

$$M = \gamma \sin \theta I_o \quad \dots\dots\dots (1)$$

Taking moment of the pressure force of surface about O:

$$M = F X_p = F (h_p / \sin \theta) \quad \dots\dots\dots (2) \quad (\sin \theta = h_p / X_p)$$

From (1) and (2),

$$\gamma \sin \theta I_o = F (h_p / \sin \theta)$$

$$h_p = (\gamma \sin^2 \theta I_o) / F = (\gamma \sin^2 \theta I_o) / \gamma h_c A$$

$$h_p = (I_o \sin^2 \theta) / (h_c A) \quad \dots\dots\dots (3)$$

But, $I_o = I_c + (A X'^2)$

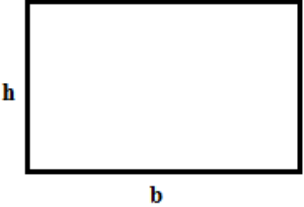
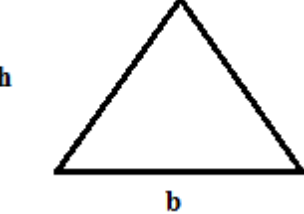
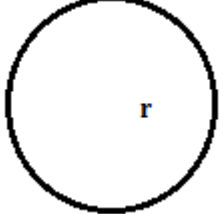
I_c : 2nd moment of area about the centre of gravity (C).

Then, in (3), $h_p = \frac{(I_c + A X'^2) \sin^2 \theta}{h_c A}$ $X' \sin \theta = h_c$

$$\therefore h_p = \frac{(I_c \sin^2 \theta)}{h_c A} + h_c$$

The metacentre is the distance between the centre of pressure and the centre of gravity. It is the term $(I_c \sin^2 \theta) / h_c A$.

The common values for I_c are illustrated in the following table.

Name	Shape	Area	Centre of Gravity, G	2 nd Moment of Area about G, I_c
Rectangle		$b h$	$h / 2$	$b h^3 / 12$
Triangle		$b h / 2$	$h / 3$	$b h^3 / 36$
Circle		πr^2	At centre	$\pi r^4 / 4$

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Example 9:

A triangular plate of 1 m base and 1.5 m height is submerged in water. The plane of the plate is inclined at 30° with the top surface of water. The base is 2 m below water surface.

- 1- Determine the total pressure on the plate?
- 2- Determine position of the centre of pressure (P)?

Solution

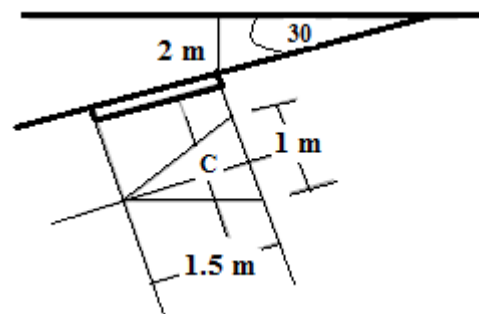
$$1- F = \gamma h_c A$$

$$\gamma = 1 \text{ t/m}^3$$

$$h_c = 2 + (0.5 \times \sin 30) = 2.25 \text{ m}$$

$$A = 0.5 \times 1 \times 1.5 = 0.75 \text{ m}^2$$

$$\therefore F = 1 \times 2.25 \times 0.75 = 1.69 \text{ t}$$



$$2- h_p = \frac{(I_c \sin^2 \theta)}{h_c A} + h_c$$

$$I_c = b h^3 / 36 = (1 \times 1.5^3) / 36 = 0.094 \text{ m}^4$$

$$h_p = \frac{(0.094 \sin^2 30)}{2.25 \times 0.75} + 2.25 = 2.264 \text{ m}$$

2-5 Total Pressure on Curved Surfaces:

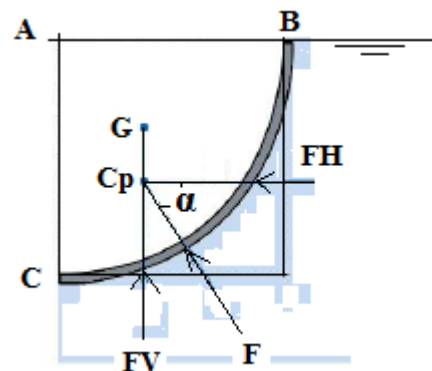
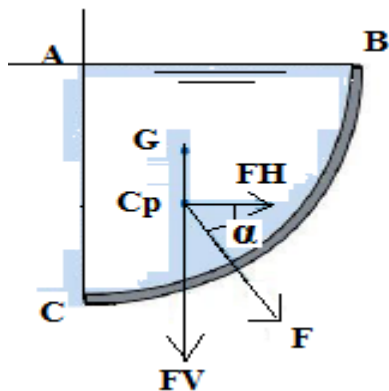
When a plane surface is submerged in a liquid, the hydrostatic forces acting perpendicularly on all elements have the same direction forming a system of parallel forces.

For the case of a curved surface, all elements do not lie in the same plane. So the hydrostatic forces (although perpendicular to their respective elements) do not form a system of parallel forces.

To get the hydrostatic force on a submerged curved surface, both horizontal and vertical components are calculated first. Then, their resultant will be the required force.

The horizontal force F_H , as shown in the figure, is the total horizontal pressure on the vertical projection of the curved surface BC. It acts at the centre of pressure of the vertical projection of the surface.

The vertical force F_V is the total weight of the liquid above the surface, in the portion ABC. It acts downward at the centre of gravity of the surface.



The magnitude of the total hydrostatic force F is:

$$F = (F_H^2 + F_V^2)^{1/2}$$

The direction of the total hydrostatic force F is:

$$\tan \alpha = F_V / F_H$$

Where α is the angle between the total force F and the horizontal.

Example 10:

A curved gate AB, as shown in figure, is a quadrant of a circular cylinder of radius 1 m.

Determine the total force on the gate per meter length?

Solution

The horizontal force:

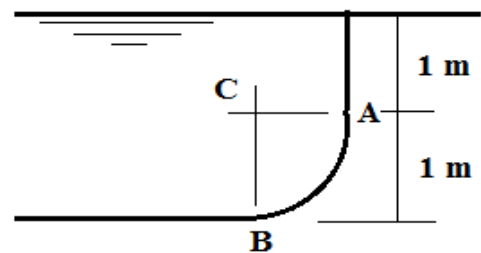
$$F_H = \gamma h_c A$$

$$\gamma = 1 \text{ t/m}^3$$

$$h_c = 1 + 0.5 = 1.5 \text{ m}$$

$$A = 1 \times 1 = 1 \text{ m}^2$$

$$\therefore F_H = 1 \times 1.5 \times 1 = 1.5 \text{ t}$$



The vertical force:

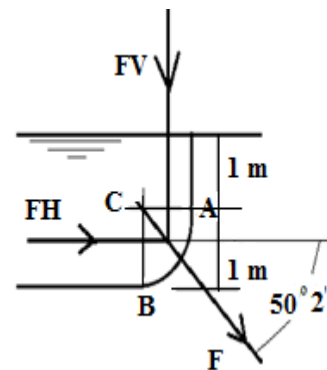
$F_V =$ Weight of water over curved surface AB

$$= \gamma \times \text{Volume} = \gamma \times A \times \text{Unit length}$$

$$\therefore F_V = 1 \times [(0.25) (\pi 2^2 / 4) + (1 \times 1)] \times 1 = 1.79 \text{ t}$$

Magnitude of the total force:

$$\therefore F = (F_H^2 + F_V^2)^{1/2} = (1.5^2 + 1.79^2)^{1/2} = 2.34 \text{ t}$$



Direction of the total force:

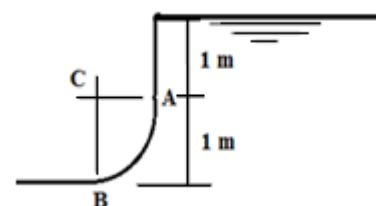
$$\tan \alpha = F_V / F_H = 1.79 / 1.5 = 1.19$$

$$\alpha = \tan^{-1} (1.19) = 50^\circ 2'$$

Where, α is the angle between the total force F and the horizontal.

Exercise:

Determine the total force on the shown gate per meter length?

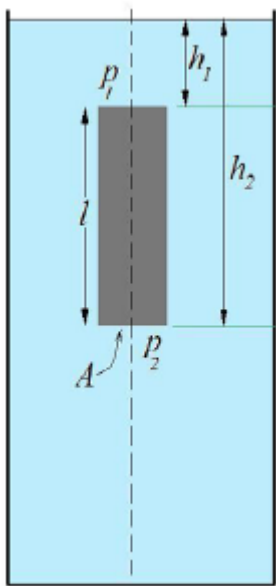


2-6 Buoyancy:

For a submerged body, there is no horizontal component for the hydrostatic force ($F_H = 0$).

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The vertical component of the hydrostatic force F_V only exists, and is called the buoyant force or upthrust F_B . This force is equal to the difference between the vertical hydrostatic forces acting on both the lower and upper surfaces of the submerged body.



$$F_B = p_2 A - p_1 A$$

$$p_1 = h_1 \rho g \text{ and } p_2 = h_2 \rho g$$

$$F_B = A(h_2 - h_1) \rho g$$

$$F_B = A l \rho g = V \rho g$$

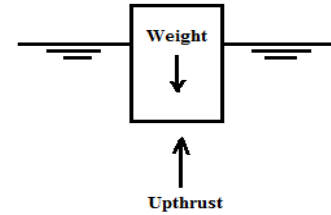
Thus the buoyant force depends only on the volume of the body (not its geometry).

It is equal to the weight of the displaced liquid and acts vertically upward at the centre of buoyancy B, which is the centre of gravity of the volume of the displaced liquid.

In other words, the centre of buoyancy is the centre of area of submerged section. This is Archimedes' principle.

In the shown figure,

- a) $F_B < W$ the body sinks to the bottom.
- b) $F_B = W$ the body floats.
- c) $F_B > W$ the body rises to the surface.



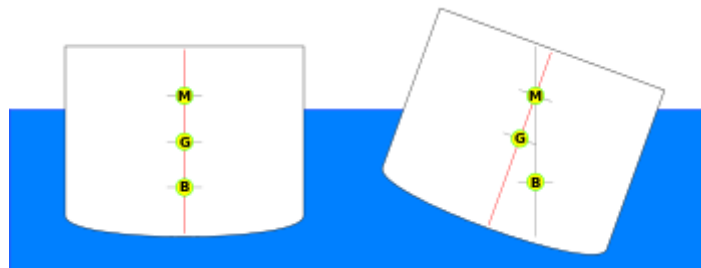
Stability of a Submerged Body:

The stability of a submerged body is detected according to the positions of both the centre of gravity (G) and the centre of buoyancy (B).

When a ship tilts, the centre of buoyancy of the ship moves laterally. It may also move up or down with respect to the water line.

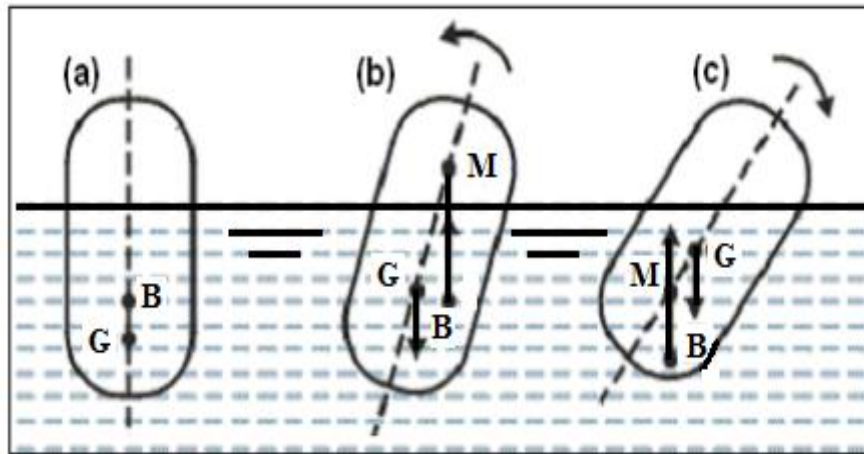
The point at which a vertical line through the tilted centre of buoyancy crosses the original centre line is the metacentre M (vertical centre of buoyancy).

The following ship stability diagram shows centre of gravity (G), centre of buoyancy (B), and metacentre (M) with ship upright and tilted over to one side.



Ship Stability Diagram

The metacentre M is considered to be fixed for small angles of tilt (usually 0-15 degrees). At larger angles of tilt, the metacentre cannot be considered fixed, and its actual location must be found to find the ship's stability.

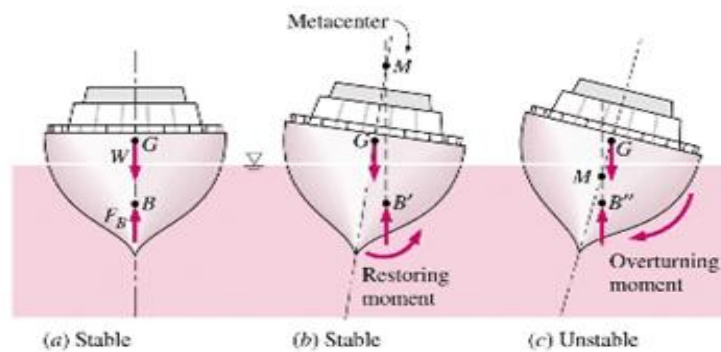
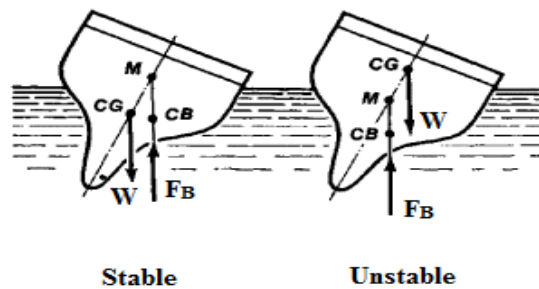


B is above G: Stable condition always.

The forces of thrust and weight are equal and inline, as shown in case (a) in the figure.

B is below G: For a small disturbance,

- M is above G is a stable condition, as shown in case (b) in the figure.
- M is below G is unstable condition, as shown in case (c) in the figure.



The metacentric height (GM) is the distance between the centre of gravity of a ship and its metacentre. It is a measurement of the initial static stability of a floating body.

- A larger metacentric height implies greater initial stability against overturning.
- The metacentric height also influences the natural period of rolling of a hull. For very large metacentric heights associated with short periods of roll, it will be uncomfortable for passengers.

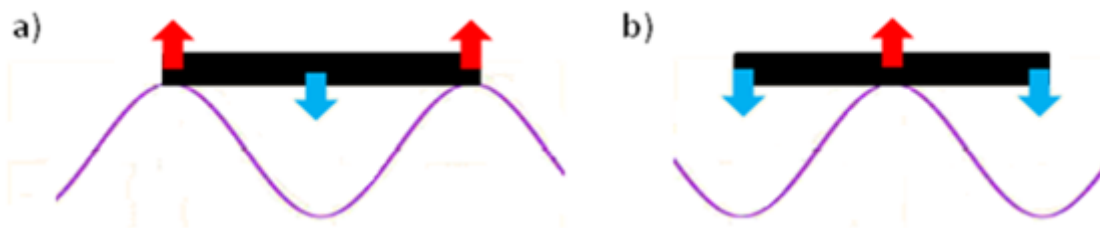
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Wavelength and Amplitude

Wavelength: The distance between two consecutive waves.

Amplitude: The height of the waves.

As shown in figure, there are two types of problems: ends of the ship at the high (a) and low (b) of the wave resulting flexure (bending).



Beam-like ships are relatively long and narrow.

The “MOL Comfort” was 316 m x 45.5 m, as shown in the following pictures.



Example 11:

A block of wood 4 m long, 2 m wide, and 1 m deep is floating in water. Specific weight of wood is 700 kg/m^3 .

- 1- Calculate the volume of displaced water?
- 2- Determine the position of centre of buoyancy?

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Solution

1- Volume of block $V = 4 \times 2 \times 1 = 8 \text{ m}^3$

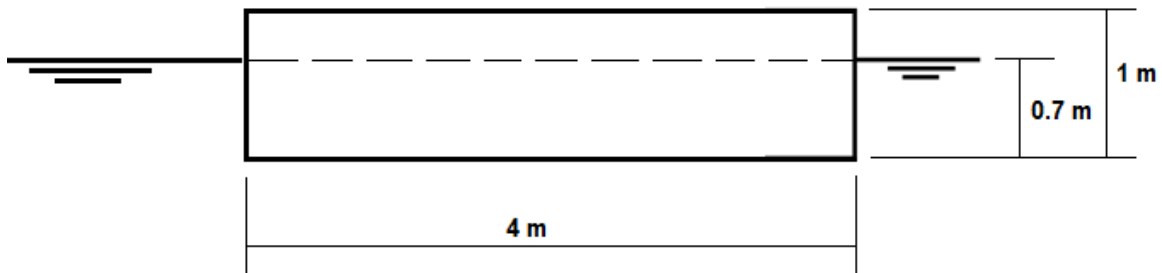
Weight of block $W = \gamma \times V = 700 \times 8 = 5600 \text{ kg}$

Floating block, $W = FB \quad m g = \gamma_w V_d \quad \gamma V_t = \gamma_w V_d$

$V_d = \gamma V_t / \gamma_w$

\therefore Volume of displaced water $V_d = 700 \times 8 / 1,000 = 5.6 \text{ m}^3$

- 2- Volume of submerged section of block = $V = 5.6 \text{ m}^3$
 = Submerged area x Submerged depth
 Submerged depth = $5.6 / (4 \times 2) = 0.7 \text{ m}$
 \therefore Centre of buoyancy $B = 0.7 / 2 = 0.35 \text{ m}$ from the base

**Example 12:**

King Hero ordered a new crown to be made from pure gold (density = 19200 kg/m^3). He asked Archimedes to check the crown. Archimedes found that the crown weighs 20.91 N when submerged in water, and the displaced water is $3.1 \times 10^{-4} \text{ m}^3$.

Is the crown made from pure gold?

Solution

$\Sigma F = W - F_B$

$20.9 = \rho_c g V - \rho_w g V = (\rho_c - \rho_w) g V$

$(\rho_c - \rho_w) = 20.9 / g V$

$\rho_c = \rho_w + 20.9 / (9.81 \times 3.1 \times 10^{-4}) = 7876 \text{ kg/m}^3 < 19200$

\therefore The crown is not made from pure gold.

Chapter 3

KINEMATICS OF FLUID FLOW

3-1 Types of Flow	3-2 The Rate of Discharge (Q)	3-3 Continuity Equation
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Kinematics of fluid flow studies the motion of the fluid without concerning the forces that cause this motion. That is to study the velocity and acceleration of fluid particles neglecting forces and energy considerations.

3-1 Types of Flow:

<u>1- Uniform Flow:</u>	<u>2- Non-Uniform Flow:</u>
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According to the effect of the distance on the flow parameters (such as velocity), the flow is uniform or non-uniform.

When the flow parameters do not change with distance, the flow is uniform.

$$\frac{dv}{dx} = 0$$

When the flow parameters change with distance, the flow is non-uniform.

$$\frac{dv}{dx} \neq 0$$

<u>3- Steady Flow:</u>	<u>4- Unsteady Flow:</u>
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According to the effect of the time on the flow parameters (such as velocity), the flow is steady or unsteady.

When the flow parameters do not change with time, the flow is steady.

$$\frac{dv}{dt} = 0$$

When the flow parameters change with time, the flow is unsteady.

$$\frac{dv}{dt} \neq 0$$

<u>5-Laminar (Streamline) Flow:</u>	<u>6- Turbulent Flow:</u>
--------------------------------------------	----------------------------------

According to the type of motion of liquid particles, the flow is laminar (streamline) or turbulent.

When the paths of liquid particles do not cross each other, the flow is laminar.	When the paths of liquid particles cross each other, the flow is turbulent.
----------------------------------------------------------------------------------	-----------------------------------------------------------------------------

<u>7- Rotational (Vortex) Flow:</u>	<u>8- Irrotational Flow:</u>
--------------------------------------------	-------------------------------------

According to the rotation of liquid particles about axes during their motion, the flow is rotational (vortex) or irrotational.

When the liquid particles rotate about axes during their motion, the flow is rotational (vortex).	When the liquid particles do not rotate about axes during their motion, the flow is irrotational.
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<u>9- One Dimensional Flow:</u>	<u>10- Two Dimensional Flow:</u>	<u>11- Three Dimensional Flow:</u>
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According to the number of directions along which the flow parameters (such as velocity) change, the flow is one or two or three dimensional flow.

When the flow parameters change in 1 direction, the flow is 1 dimensional flow	When the flow parameters change in 2 directions, the flow is 2 dimensional flow	When the flow parameters change in 3 directions, the flow is 3 dimensional flow
--------------------------------------------------------------------------------	---------------------------------------------------------------------------------	---------------------------------------------------------------------------------

3-2 The Rate of Discharge (Q):

It is the quantity of liquid flowing through a section of the conduit per unit time.

$$Q = V / t = (A l) / t = A \times (l / t)$$

$$Q = A v$$

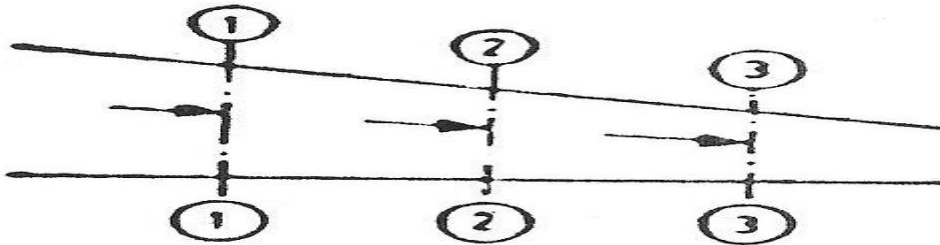
Where, Q: The discharge (rate of flow).

A: The cross sectional area of the conduit.

v: The average velocity of liquid.

3-3 Continuity Equation:

Consider a liquid flows through a conduit, as shown in figure, and the values of cross sectional area, velocity and density of liquid are A_1 , v_1 , ρ_1 and A_2 , v_2 , ρ_2 and A_3 , v_3 , ρ_3 for the three sections respectively.



From the principle of conservation of mass, mass of liquid flowing per unit time through section 1 = mass of liquid flowing per unit time through section 2 + change of mass of liquid per unit time between sections 1 & 2 = mass of liquid flowing per unit time through section 3 + change of mass of liquid per unit time between sections 2 & 3.

For a steady flow, there is no change of liquid parameters with time. There is no change in mass of liquid with time. Thus, mass of liquid flowing per unit time through section 1 = mass of liquid flowing per unit time through section 2 = mass of liquid flowing per unit time through section 3.

$$M_1 / t = M_2 / t = M_3 / t$$

But, $M / t = \rho V / t = (\rho A l) / t = \rho \times A \times (l / t) = \rho A v$

Then, $\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \rho_3 A_3 v_3$

Because liquids are incompressible fluids, the density ρ is constant.

$$A_1 v_1 = A_2 v_2 = A_3 v_3 = \text{Constant}$$

$$Q_1 = Q_2 = Q_3 = \text{Constant}$$

Example 1:

Water flows through a pipe of 10 cm diameter with a velocity of 10 m/sec.

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1- Determine the discharge?

2- If the diameter of the pipe is 20 cm, determine the velocity?

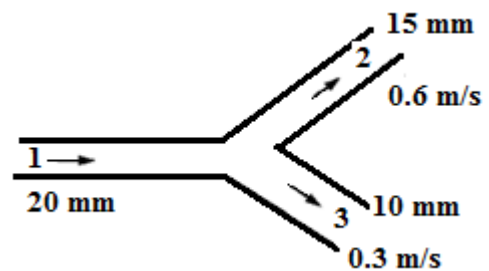
Solution

$$1- \quad Q = A v = (\pi (0.1)^2 / 4) \times 10 = 0.078 \text{ m}^3/\text{sec}$$

$$2- \quad v = Q / A = 0.078 / (\pi (0.2)^2 / 4) = 2.48 \text{ m/sec}$$

Example 2:

As shown in figure, water flows through the pipe (1) that branches into two pipes (2) and (3).



1) Determine the rate of flow in cm^3/s for the pipe (1)?

2) Find the velocity in m/s for the pipe (1)?

Solution

$$1) \quad Q_1 = Q_2 + Q_3 = (A_2 v_2) + (A_3 v_3)$$

$$\begin{aligned} Q_1 &= \{(\pi (1.5)^2 / 4) * 60\} + \{(\pi (1)^2 / 4) * 30\} \\ &= 129.6 \text{ cm}^3/\text{sec} \end{aligned}$$

$$2) \quad v_1 = Q_1 / A_1 = 129.6 / (\pi (2)^2 / 4)$$

$$\begin{aligned} v_1 &= 41.25 \text{ cm/sec} \\ &= 0.41 \text{ m/sec} \end{aligned}$$